Development of simultaneous energy and reserve dispatch model and corresponding pricing mechanism

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1. Abstract

This study aims to develop a market structure accounting for the energy and reserve capacity procurement. The proposed market scheme is constituted of two main parts: a 5-min simultaneous energy and reserve pre-dispatch model and re-dispatch model for the real operating time of the power system. This market design is coupled with a contingency analysis methodology pursuing to identify the robustness of the power system under various operational conditions.

The following report is organized as follows. The first part is an introduction to the subject of ancillary services discussing their importance for the reliable operation of the power system and provides the motivation for this work. In the second part proper definitions for the different types of ancillary services are given. Furthermore, a detailed description of the different ancillary services market schemes is presented.

The third part of the report provides the theoretical background and the mathematical formulation of the 5-min simultaneous energy and reserve pre-dispatch model and the re-dispatch model developed within the framework of this study. In addition, the methodology used for the contingency analysis is analyzed.

The fourth part includes the description of a case study and the presentation and discussion of the results obtained from the implementation of the developed market structure in cases of contingencies.

Finally, the last part of the report draws some overall conclusions and suggests possible ideas about further work.
# Table of Contents

1. Abstract ........................................................................................................................................... 2  
2. Introduction and Motivations ........................................................................................................... 4  
3. Ancillary Services definitions and types of reserve market schemes ............................................ 6  
   3.1 Definitions ..................................................................................................................................... 6  
   3.2 Types of reserves market schemes .............................................................................................. 8  
      3.2.1 Sequential Auction of Ancillary Services Model ................................................................. 8  
      3.2.2 Simultaneous Energy and Ancillary Services Model .......................................................... 11  
4. Simultaneous energy and reserve pre-dispatch model and energy re-dispatch model for 5 min real time market ...................................................................................................................... 21  
5. Case Study ........................................................................................................................................ 27  
   5.1 Description of the power system ................................................................................................. 27  
   5.2 Results ......................................................................................................................................... 30  
6. Conclusion ........................................................................................................................................ 39  
7. Future work ....................................................................................................................................... 39  
8. References ......................................................................................................................................... 40  
9. Acknowledgements .......................................................................................................................... 40
2. Introduction and Motivations

The operation of the power systems is inherently stochastic since all their main components e.g. load and power generation are evolving dynamically in respect to time. All the TSOs are using forecasting tools regarding the demand and the available power production in order to plan effectively the operation of the power system. However, during the real time operation several deviations from the initial planning may occur either in demand side or in the generation side. Therefore, the development of efficient technical and fiscal tools able to handle the imbalance of demand and supply should always be considered.

One of the key ingredients of the modern power systems are the Ancillary Services (A/S), which are used to keep or to restore the system’s characteristics i.e. frequency within their nominal values. Therefore, Ancillary Services play an important role in the electricity market as far as their procurement is critical for the reliable and economic operation of the power system. The term A/S usually includes the regulation – up and –down, the spinning reserves and the non-spinning reserves. In a deregulated energy market which faces also a high penetration of partly controllable renewable energy sources and distributed power production, an economically efficient pricing mechanism of A/S is needed in order to properly reward the A/S providers and maintain the balancing between procurement costs of the system operator and the revenues of the providers, while ensuring the secure operation of the power system.

Currently, there are two main categories of reserve markets schemes, each one with different characteristics, advantages and disadvantages. The first category includes market designs where the procurement of ancillary services is achieved through sequential auctions. In the second category, the so called integrated systems are included, where a simultaneous energy and reserves capacity is performed. In the following parts of this report the detailed definitions of different types of A/S are given. In addition, a thorough description of the different market structures is presented along with their proper mathematical formulation.

The main motivation for this work is to develop a market structure for reserves which will operate very close to real time and provide valuable information to the TSO taking into account the existing state of the power system including any contingencies that may have occurred.

The proposed model, which is described extensively in the following parts of the report, is constituted from two main parts. The first accounts for the simultaneous energy and reserves dispatch in discrete times \( T \). The second part consists of a re-dispatching algorithm, based on an Optimal Power Flow (OPF) model which is
executed during the real time of the operation. Both parts are formulated such that to take into consideration the actual operational conditions of the power system. The time step between the discrete times $T$ is set equal to 5 minutes.

The considered market structure is used in combination with a contingency analysis methodology in order to identify possible load shedding situations and the corresponding cost of the power system under various contingency states. The algorithm used for coupling the above mentioned components is explained latter in this report.

Finally, a case study in a small power system is constructed and the results from applying the methodology described are provided analytically. The study of those results may provide important information about the ability of the power system to respond effectively in different contingency situations while its total cost optimization is pursued.
3. Ancillary Services definitions and types of reserve market schemes

3.1 Definitions

This part of the report provides the basic definitions for the different types of ancillary services, as well as an overview of the main types of reserve market types.

A clear and coherent definition of the ancillary services is the following:

“Ancillary Services can be defined as services other than energy that are essential for ensuring the reliable operation of the electrical grid “. [1]

As far as the operation of the power system is dynamic and possible contingencies may occur over time e.g. generator outage or unexpected load variations, the ancillary services in a form reserve capacity they should be able to compensate adequately the load. The following figure depicts the operation of the power system over different time scales.

![Figure 1 Timescales for Utility Operations][3]

There are four main types of reserves namely frequency response reserves, spinning and non-spinning reserves and replacement reserves. The first type is used to follow the load variations over time and adjust the output of the power resources in order to keep the frequency of the power system in the nominal value e.g. 50 or 60 Hz. The spinning and non-spinning reserves account for unloaded generating capacity and they responsible for providing the necessary additional generation in cases of a contingency such as a loss of a generator or a significant increase of the load. The main difference between these types of reserves is that spinning reserves are operating below their nominal output and therefore they are able to contribute
instantly by increasing their production. On the other hand, the non-spinning reserves are not on-line but they should be able to generate capacity for emerging condition in a quite short time e.g. 10 minutes. Finally, the replacement reserves are obliged to be able to produce power within 60 minutes either from on-line or off-line generating capacity, interruptible load or imports. [1] [2]

The different reserves types may also be categorized according to the time horizon that it is needed for their activation in respect to the real time operation of the power system. Hence, we may the primary reserves that it consists of units equipped with Automatic Generation Control (AGC) and are used to minimize the deviations of the frequency and restore the equilibrium between supply and demand. The secondary reserve includes those generating units that they can activated either manually or automatically and are used in order to restore the power balance and bring the frequency back to its nominal value. Finally, the term of tertiary reserve refers to the power that can be manually or automatically activated in order to provide or restore an adequate secondary control reserve and provide a permanent solution. [2] [3]

The following figure provides an example of activation of different reserves types in case of a unit outage. It can be observed that in this case the primary reserves are used in the first place in order to stabilize the frequency of the power system. Later on the secondary reserves are activated and they restore the frequency back to its nominal value e.g. 50 Hz. Finally, the tertiary reserves (long-term reserve) take over in order to restore the secondary reserves.

\[\text{Figure 2 Activation of Power Reserves and Frequency of Power System as a Function of Time when a large Power Plant is disconnected from the Power System.} [3]\]
3.2 Types of reserves market schemes

Currently around the different regional energy markets there are two main types of reserve market schemes. The first is called sequential auction of ancillary services and the second is simultaneous auction of energy and reserves.

3.2.1 Sequential Auction of Ancillary Services Model

The first type of reserves market consists of sequential central auctions operated by the TSO. In this case after the closure of energy market and transmission market for congestion management the different types of ancillary services are procured by the TSO through the ancillary services market as shown in Figure 3.

The main advantage of this type of market is its relatively simple form which does not ask for sophisticated optimization methods [4].

Under this category there might be two different models of market structure.

The first model does the reserves scheduling taking into account only the capacity bid for the different reserves ($/MW), while the energy bid ($/MWh) is not taken into account during the optimization procedure of reserves procurement but only when the compensation of ancillary service providers is considered. [2]

The mathematical formulation of this model is the following:

\[
\min \sum_{i=1}^{N} C^R(i,t) \cdot RQ(i,t)
\]

Subject to

\[
\sum_{i=1}^{N} RQ(i,t) \geq RQ^{eq}(t)
\]

\[
RQ(i,t) \leq RQ^{Max}(i,t)
\]

Where the following notation is used
Capacity bid ($/MW)

Reserve Quantity procured (MW)

Reserve Requirement (MW)

Maximum available Reserve Quantity (MW)

The indices \( i, t \) denote the units and the time respectively.

In this case the market clearing price for ancillary services is equal to the capacity bid for the most expensive ancillary services provider accepted in the auction.

This type sequential auction of ancillary services has the important disadvantage that the TSO is exposed to significant risk. This is because in case when reserves with low capacity costs but high energy bids are activated the TSO will face significantly high payments to the providers of those reserves.

One possible way in order to mitigate the risk of a TSO, as described previously, who operates a sequential auction of ancillary service, is to include to the existing model the expected energy usage as well as the energy bids from the various reserves. [2]

In this case the optimization model would have the following form:

\[
\min \sum_{i=1}^{N} [C^R(i, t) + x \cdot C^E] \cdot RQ(i, t)
\]

Subject to

\[
\sum_{i=1}^{N} RQ(i, t) \geq RQ^{req}(t)
\]

\[
RQ(i, t) \leq RQ^{Max}(i, t)
\]

Where the following additional symbols are used:

\( C^E \)  
Energy Bid ($/MWh)

\( x \)  
Expected Energy Usage

In this case the market clearing price for ancillary services is calculated as:

\[
MPC = \max [C^R(i, t) + x \cdot C^E]
\]
It should be noted that this model does not account for the substitution of higher quality services for a lower one. For example, regulation reserve to be used as spinning reserve or spining reserve to be used as non-spinning etc.

This type of sequential auction of ancillary services is widely used from many TSOs around the world such as in Australia, Britain and the Nordic Power Market where Denmark is also included.

The current Nordic electricity market consists of a number of specific underlying markets based on a timeline for the bidding offers. The following figure shows the most important part of this market. [5]

![Figure 4 Different markets for different time regimes – the Nordic set-up. The reservation markets include reservation of resources for the regulating power market and other reserves [10]](image)

One of the main objectives of the Danish TSO – Energinet.dk is to maintain the balance between the power supply and demand during the operation of the power system. This is accomplished by utilizing different types of reserves.

In order to have stability in the Nordic electricity system different criteria set by the TSO associations must be met at all times [6]:

1. The frequency of the synchronous system must be between 49.9 and 50.1 Hz
2. The time deviation of the synchronization shall be within the range [-30s, 30s]. Time deviation is found by integrating the frequency deviation from 50 Hz
3. The requirement in RG Continental Europe that every control area has to keep its own balance
4. The transmission capacity must not be exceeded at any line.
Any imbalances caused in the system due to deviation of demand from the actual energy production are initially covered by the automatic reserves, which are purchased by the TSO. The TSO (Energinet.dk) buys automatic reserves according to their type and it pays for their reserve capacity and the energy production in case they are activated, as described in the previous models. The automatic reserves are in general expensive and have limited capacity.

In order to reduce the payments for automatic reserves, the TSO uses regulating power, which is a manually activated reserve and is determined as increased or decreased generation that can be fully activated within 15 minutes.

In the Nordic Power Market all the involved TSOs operate a common regulating power market. The units that are able to provide regulating power send their bids, which consist of reserve capacity (MW) and the corresponding price (DKK/MWh). Shorting those bids in a decreasing order the NOIS-list is created, where prices for up-regulation are above the spot price and prices for down-regulation are below spot price. Using this list, during the real time operation the TSO is able to activate the reserve with the lowest price, according to the needs of the power system and the technical constraints imposed.

Until now this market structure seems to operate effectively, as far as the reserve prices are kept low. It should be noted that one essential factor for the efficient operation of this market up to date, is the significant capacity of the hydro power plants operating in Norway, which are able to adjust their production in a short time in order to follow the load consumption.

However, one can mention also some important disadvantages of the existing regulating power market which must be taken into account during the upcoming period as the power system will move towards higher penetration of renewable energy sources e.g. wind power which are partly controllable and have a stochastic production.

The most essential drawback of the current market structure is the lower limit of the reserve capacity bid e.g. 10 MW that a unit may apply. As a result consumers and small scale distributed power generation is not eligible to participate in the regulating market. Furthermore, some technical barriers related with the real-time measurement of consumption and production should be overcome. [5]

3.2.2 Simultaneous Energy and Ancillary Services Model

The second type of reserves procurement model is based on an integrated model which dispatches simultaneously the energy and the reserves capacity. Using this type of models the TSO is able to optimize the total cost of generation and reserves
procurement while meeting the technical constraints of the power system e.g. transmission capacity etc.

Despite that such kind of market structure is more complex than the sequential auction of ancillary services; it may produce better price signals that contain information about the opportunity costs of scarce resources over the whole power system. This type of market structure is used, among others, by TSOs in New York, New England and PJM as well as in Britain from 1989 to 2001. [1]

In the general case the optimization model followed by the TSOs that are using this market structure has the formulation presented below. The formulation of simultaneous energy and ancillary services dispatch presented here is used by the NYISO and it is discussed thoroughly in [1].

This model for simultaneous energy and reserve procurement is based on the AC optimal power flow (AC-OPF) algorithm, and produces the needed locational marginal prices (LMP) of energy and the ancillary service marginal prices (ASMP), for the considered A/S types. Furthermore, the execution of the AC-OPF provides the optimal dispatch of power and A/S procurement capacity from the committed generation, while satisfying the AC power flow equations, the A/S requirements and other transmission and operating constraints. The formulation presented below, refers to normal operating conditions of the power system. Otherwise, simulation of contingency scenarios (N-1) could be used in order to define any additional operating constraints that may arise.

The objective of the AC-OPF is to minimize the total system cost regarding both the energy and the A/S procurement, over the considered time interval (e.g. 1 hour). The energy cost functions are considered to be piece-wise linear convex curves, while the A/S cost function is linear representing a constant bid price. The analytical mathematical expression of the objective function is:

\[
C^{Total} = \sum_{i=1}^{N-1} C_i(P_i) + C_N[P_N(x)] + \sum_{i \in I_{RU}} C_i^{RU}(RU_i) \\
+ \sum_{i \in I_{SP}} C_i^{SP}(SP_i) + \sum_{i \in I_{NS}} C_i^{NS}(NS_i) + \sum_{i \in I_{RD}} C_i^{RD}(RD_i)
\]

Where the following notation is used:

\[
C^{Total} \quad \text{Total cost of energy and A/S}
\]

\[
C_i(P_i) \quad \text{Energy cost function at bus i}
\]

\[
C_N[P_N(x)] \quad \text{Energy cost function at reference bus}
\]
In the meanwhile the following constraints should be met at any given time of the operation of the power system.

- Power Balance Constraint

The power balance constraints are described by ac power flow equations, for the active and the reactive power respectively, as:

\[
\Delta P_i(x, P_i) = P_i(x) - P_i = P_{G_i} - P_{D_i} - \sum_{k=1}^{n} |V_i||Y_{ik}| |V_k| \cos(\theta_i - \theta_k - \phi_{ik}) = 0
\]

for \(i = 1 \ldots N - 1\)

\[
\Delta Q_i(x, Q) = Q_i(x) - Q_i = Q_{G_i} - Q_{D_i} - \sum_{k=1}^{n} |V_i||Y_{ik}| |V_k| \sin(\theta_i - \theta_k - \phi_{ik}) = 0
\]

for \(i = 1 \ldots N_d\)

Where the following enumeration is used:
• $i = 1 \ldots N_d$ for PQ buses (i.e. load/generator operating at reactive power limit)
• $i = N_d + 1 \ldots N_d + N_g$ for PV buses (i.e. generation or load with voltage control)
• $i = N$ for the slack bus (reference)

$x = [\theta_1, \theta_2 \ldots, \theta_{N-1} \ V_1, V_2 \ldots, V_{N-1}]^T$ represent the voltage phase angles $\theta_i$ and magnitudes $V_i$. The voltage phase angle $\theta_N$ at the reference bus is set equal to zero.

Furthermore, the active power losses of the system can be determined as:

$$\sum_{i=1}^{N} P_i(x) - P_{loss} = 0$$

• Capacity Reserves Constraint

The capacity reserve constraints are inequality constraints that used to ensure the solution will meet the system operator requirements, based on load forecast and other operating system conditions.

The following equations specify the total amount of each A/S that need to be procured by the system operator. Furthermore, they are taking into account that a higher quality A/S might be procured in order to meet the requirement of a lower quality service. Hence, the following substitutions are allowed:

1. Regulation up can meet spin and non-spin requirements
2. Spin can meet non-spin requirements

Regulation-Up Requirement

$$R^{RU} - \sum_{i \in \mathcal{R}_U} RU_i \leq 0$$

Spinning Reserve Requirement

$$R^{RU} + R^{SP} - \sum_{i \in \mathcal{R}_U} RU_i - \sum_{i \in \mathcal{SP}} SP_i \leq 0$$

Non- Spinning Reserve Requirement

$$R^{RU} + R^{SP} + R^{NS} - \sum_{i \in \mathcal{R}_U} RU_i - \sum_{i \in \mathcal{SP}} SP_i - \sum_{i \in \mathcal{NS}} NS_i \leq 0$$

Regulation-Down Requirement

$$R^{RD} - \sum_{i \in \mathcal{RD}} RD_i \leq 0$$
The equations below are used to ensure that the awarded capacity for each A/S would be non-negative and in the meantime lower than the corresponding upper limit referring either on the bid limit or any physical boundaries of the generators.

\[ 0 \leq RU_i \leq RU_i^{Max} \quad \text{Regulation Up Bid Limit} \]
\[ 0 \leq SP_i \leq SP_i^{Max} \quad \text{Spinning Bid Limit} \]
\[ 0 \leq NS_i \leq NS_i^{Max} \quad \text{Non-Spinning Bid Limit} \]
\[ 0 \leq RD_i \leq RD_i^{Max} \quad \text{Regulation Up Bid Limit} \]

Regarding the Non-Spin Bid quantity it should be mentioned that despite it is a continuous variable from zero to the upper limit, in case the considered unit becomes on-line, it will have to produce its minimum generation.

In the equations presented above the \( R \) variable denotes the requirement for the considered A/S.

- **Supply Constraints**

The total active power output of each generator is bounded by the following limits:

\[ P_i + RU_i + SP_i + NS_i - P_i^{Max} \leq 0 \quad \text{Active Power Maximum Limit} \]
\[ P_i^{Min} - P_i + RD_i \leq 0 \quad \text{Active Power Minimum Limit} \]

Ramp-Up Limit \( \frac{RU_i}{RR_i^{RU}} + \frac{SP_i + NS_i}{RR_i^{OP}} - T_{ramp} \leq 0 \) Ramp-Up Limit

Where \( P_i^{Max} \) and \( P_i^{Min} \) denote the maximum and the minimum operating limits of each resource at bus. In addition, \( RR_i^{RU} \) and \( RR_i^{OP} \) indicate the ramp rate [MW/min] and \( T_{ramp} \) the time interval for the desired ramp (e.g. 10 min).

- **Network Constraints**

The network constraints in the considered formulation may include:

- The reactive power supply limits
- The voltage magnitude and phase angle limits
- The branch flow limits

All above mentioned constrained can be written in the following form:

\[ F_k(x) - F_k^{Max} \leq 0 \]

Where \( F_k(x) \) is the quantity limited by the constraint \( k \) and \( F_k^{Max} \) the corresponding upper limit.
An illustrative example of the network constraint formulation regarding the voltage magnitudes in the PQ buses is:

\[ V_i - V_i^{Max} \leq 0 \text{ for } i = 1, \ldots, N_d \]
\[ V_i^{Min} - V_i \leq 0 \text{ for } i = 1, \ldots, N_d \]

Where \( V_i^{Min} \) and \( V_i^{Max} \) are the lower and upper bounds of the voltage magnitude at bus \( i \).

In order to calculate the prices and the components for the energy and the ancillary services on each bus of the power system the Lagrange function of the above presented formulation is constructed as:

\[
L = \sum_{i=1}^{N-1} C_i(P_i) + C_N[P_N(x)] \quad \text{Energy Cost}
\]

\[ + \sum_{i \in I_{RU}} C_i^{RU}(RU_i) \quad \text{Reg-up Cost} \]

\[ + \sum_{i \in I_{SP}} C_i^{SP}(SP_i) \quad \text{Spin Cost} \]

\[ + \sum_{i \in I_{NS}} C_i^{NS}(NS_i) \quad \text{Non-Spin Cost} \]

\[ + \sum_{i \in I_{RD}} C_i^{RD}(RD_i) \quad \text{Reg-down Cost} \]

\[ + \sum_{i=1}^{N-1} \lambda_i[P_i(x) - P_i] \quad \text{Active Power Balance} \]

\[ + \sum_{i=1}^{Nd} \gamma_i[Q_i(x) - Q_i] \quad \text{Active Power Balance at PQ buses} \]

\[ + \sum_i \lambda_i^{RU}(R^{RU} - \sum_{i \in I_{RU}} RU_i) \quad \text{Reg-up Requirement} \]

\[ + \sum_i \lambda_i^{SP}(R^{RU} + R^{SP} - \sum_{i \in I_{RU}} RU_i - \sum_{i \in I_{SP}} SP_i) \quad \text{Spin Requirement} \]
\[
\begin{align*}
\sum_{i} \lambda_i^{NS} (R_i^{RU} + R_i^{SP} + R_i^{NS}) & \quad \text{Non-Spin Requirement} \\
+ \ & \sum_{i \in RU} \left( RU_i - \sum_{i \in SP} SP_i - \sum_{i \in NS} NS_i \right) \\
+ \sum_{i} \lambda_i^{RD} (R_i^{RD} - \sum_{i \in RD} RD_i) & \quad \text{Reg-Down Requirement} \\
+ \ & \sum_{i=1}^{N-1} \pi_i^{max} (P_i + RU_i + SP_i + NS_i - P_i^{Max}) \\
+ \ & \sum_{i=1}^{N-1} \pi_i^{min} (P_i^{Min} - P_i + RD_i) \\
+ \sum_{i \in RU} a_i^{RU} (RU_i - RU_i^{Max}) & \quad \text{Max Reg-Up Limit} \\
+ \ & \sum_{i \in RU} \beta_i^{RU} (-RU_i) \quad \text{Min Reg-Up Limit} \\
+ \sum_{i \in SP} a_i^{SP} (SP_i - SP_i^{Max}) & \quad \text{Max Spin Limit} \\
+ \ & \sum_{i \in SP} \beta_i^{SP} (-SP_i) \quad \text{Min Spin Limit} \\
+ \sum_{i \in NS} a_i^{NS} (NS_i - NS_i^{Max}) & \quad \text{Max Non-Spin Limit} \\
+ \ & \sum_{i \in NS} \beta_i^{NS} (-NS_i) \quad \text{Min Non-Spin Limit} \\
+ \sum_{i \in RD} a_i^{RD} (RD_i - RD_i^{Max}) & \quad \text{Max Reg-Down Limit} \\
+ \ & \sum_{i \in RD} \beta_i^{RD} (-RD_i) \quad \text{Min Reg-Down Limit} \\
+ \sum_{i=1}^{N-1} a_i^{OP} \left( \frac{RU_i}{RR_i^{RU}} + \frac{SP_i + NS_i}{RR_i^{OP}} - T_{ramp} \right) & \quad \text{Ramp Up Limit} \\
+ \sum_{k} \mu_k (F_k(x) - F_k^{Max}) & \quad \text{Network Constraints}
\end{align*}
\]
The Greek letters denote the Lagrange multipliers of the corresponding constraints.

**Local Marginal Prices (LMPs) for energy**

The LMP can be defined as the cost of supplying an increment of load at a particular location. In other words, LMP is the change of the total production cost to deliver an additional increment of load to a location. The LMPs are products of AC-OPF and can be calculated as:

\[
\frac{\partial L}{\partial \Delta P_i} = \frac{\partial L}{\partial [P_i(x) - P_i]} = \lambda_i
\]

Where \( \lambda_i \) is corresponding LMP at bus \( i \).

Each nodal price (LMP) can be decomposed into three main components, namely i) the incremental cost at the reference bus ii) the incremental cost of the transmission losses and iii) the incremental cost of the network constraints, which include the transmission constraints, power supply constraints, voltage constraints and phase angle constraints. As a result LMP can be written as:

\[
\lambda_i = \lambda_N - \lambda_i L_i - \sum_k \mu_k S_{ki}
\]

Where

- \( \lambda_N = \frac{\partial C_N}{\partial P_N} \) the system marginal cost at the reference bus
- \( L_i = \frac{\partial P_{\text{loss}}}{\partial P_i} \) the \( i \)th loss contribution factor
- \( S_{ki} = \frac{\partial f_k}{\partial P_i} \) the sensitivity of the quantity limited by constraint \( k \) with respect to the active power injected into node \( i \) and withdrawn at the reference bus.

Especially in case the binding constraint \( k \) refers to transmission capacity limits the quantity \( S_{ki} \) can be seen as the generation shift factor, defined as [10]:

\[
a_f = \frac{\Delta f_i}{\Delta P_i}
\]

Where \( l = \) the line index

\( i = \) the bus index

\( \Delta P_i = \) change in generation at bus \( i \)
\( \Delta f_i = \text{change in active power flow on line } l \text{ when a change in generation, } \Delta P_i, \text{ occurs at bus } i \)

This formulation assumes that any change in generation, \( \Delta P_i \), is picked by the reference bus, while the output of the remaining generators remains the same. The generation shift factor \( a_{fi} \), represents the sensitivity of the flow line \( l \) to change in generation at bus \( i \).

It should be mentioned that despite that the LMP is indifferent from the choice of the reference bus, the loss and transmission sensitivity components depend on the selection of the reference bus.

**Ancillary Service Marginal Prices (ASMP)**

The ASMP represents the incremental cost of providing one additional unit of A/S into the power system. The ASMP of each A/S type are calculated as:

\[
\frac{\partial L}{\partial R^{RU}} = \lambda_t^{RU} + \lambda_t^{SP} + \lambda_t^{NS} \quad \text{Regulation-Up Price}
\]

\[
\frac{\partial L}{\partial R^{SP}} = \lambda_t^{SP} + \lambda_t^{NS} \quad \text{Spinning Price}
\]

\[
\frac{\partial L}{\partial R^{NS}} = \lambda_t^{NS} \quad \text{Non-Spinning Price}
\]

\[
\frac{\partial L}{\partial R^{RD}} = \lambda_t^{RD} \quad \text{Regulation-Down Price}
\]

When a higher quality A/S might be procured in order to meet the requirement of a lower quality service, the ASMPs for the two services are equal to the price of lower quality service. This property could be illustrated in case regulation-up is used to meet the spin requirement.

Then

\[
R^{RU} - \sum_{i \in i \in R^{RU}} RU_i \leq 0
\]

Since \( \sum_{i \in i \in R^{RU}} RU_i = R^{RU} + \alpha \cdot R^{SP} \), where \( \alpha \) is the percentage of the spin requirement covered by the regulation-up capacity.

The following KKT condition applies:

\[
\lambda_t^{RU} (R^{RU} - \sum_{i \in i \in R^{RU}} RU_i) = 0 \Rightarrow \lambda_t^{RU} = 0
\]
Consequently:

\[
\frac{\partial L}{\partial R_{RU}} = \frac{\partial L}{\partial R_{SP}} = \lambda_i^{SP} + \lambda_i^{NS}
\]

To conclude with, there are various types of reserve markets operated by the TSOs around the world. Each market design has its advantages and drawbacks. Therefore, each TSO should be chose among them the one which serves his needs more suitably.

The sequential reserves procurement market scheme is less complex in terms of optimization formulation, but it may produce undesirable results for the TSO either due to high energy bids of some reserves or due to its inability to allow substitution between the different reserves according to their quality.

On the other hand, using a simultaneous energy and reserves dispatch model may ask for more sophisticated formulation of the problem. However, it gives the ability to the TSO to account for the quality of reserves he procures and therefore avoid high risks associated with the cost bids of higher or lower quality reserves.

A detailed overview and assessment of different reserve market schemes is presented in [7].
4. Simultaneous energy and reserve pre-dispatch model and energy re-dispatch model for 5 min real time market.

The main idea of this project is to develop a new market structure where the information about the prices of energy and reserves will be updated within a short time frame. According to the proposed market scheme, a simultaneous energy and reserves dispatch optimization problem will be solved every certain time periods e.g. \( t=5 \) minutes, during the continuous operation of the power system. As a result, the LMPs and the ASMPs would be obtained in every discrete time step \( T \). Those prices can be noted as Ex-Ante prices. In a further step and during the real time operation of the power system the Ex-Post prices can be calculated by re-dispatching the system according to results of the simultaneous energy and reserves dispatch obtained from at the last discrete time step \( T \).

This market model may be applied in the field of contingency analysis of the power system in order to evaluate its robustness and the short term operating cost. The aim of following part is to describe a general methodology which provides the main technical and economic parameters of the power system under different contingencies e.g. unit outage, during the real operation time. The results of such a study have high value for the TSO and all the other stakeholders involved in the operation of the power system, since they can provide useful information about its efficiency, both in technical and economic terms, as well as signals regarding the future development and reinforcement of the existing infrastructure.

The methodology described in this part focuses only on the regulation – up reserves and does not consider the rest of the reserves types presented above e.g. regulation down, spinning and non-spinning reserves. However, the current methodology can be easily generalized in order to include further reserve types as described thoroughly in the previous parts.

This market structure may yield multiple benefits for the TSO and consequently for all the stakeholders involved in the operation of the power system i.e. customers, generation companies. The main advantage of this method is that it uses up to date information e.g. very close to real time, about the operational conditions of the power system. Therefore, it allows the TSO for better planning and provides him with the ability to confront effectively any emergency situations that may arise e.g. contingencies.

The following figure presents schematically the proposed structure of energy and reserves market, where it is also shown how and when the Ex-Ante and Ex-Post prices are calculated.
The simultaneous energy and reserves dispatch optimization model used here in every discrete time step $T$ and for different operational states $j$ of the power system is summarized by the following equations:

$$\min C_{T,j}^{total} = \sum_{i=1}^{N-1} C_i(P_{i,T,j}) + C_N[P_{N,T,j}(x_{T,j})] + \sum_{i \in RU} C_i(RU_{i,T,j})$$

Subject to:

$$\Delta P_{i,T,j}(x_{T,j}, P_{i,T}) = P_i(x_{T,j}) - P_{i,T,j} = 0$$

$$\Delta Q_{i,T,j}(x_{T,j}, Q_{T,j}) = Q_i(x_{T,j}) - Q_{LT,j} = 0$$

$$RU_{T,j} - \sum_{i \in RU} RU_{i,T,j} \leq 0$$

$$0 \leq RU_{i,T,j} \leq RU_{i,T,j}^{Max}$$

$$P_{i,T,j} + RU_{i,T,j} - P_{i,T,j}^{Max} \leq 0$$

$$P_i^{Min} - P_{i,T,j} \leq 0$$

$$F_{k,T}(x_{T,j}) - F_{k,T}^{Max} \leq 0$$

$$RU_{T,j} = \min \left( \max\left(\rho_i^{Max}, \sum_{i \in ol} P_i^{Max} - \sum_{i} P_{DT,j} - P_{Loss,T,j}^{Max} \right) \right)$$
The notation and the symbols are the same as described in part 3.2.2.

It should be noted that in addition to the existing generators of the power system an equal number of “imaginary” generators are added to the above presented simultaneous energy and reserves dispatch model in order to simulate the load shedding situations. Each of these generators faces an extremely high power production cost which could set equal to the Value of Lost Load (VOLL) as determined by the TSO. [8]

Furthermore, their lower production limits are set to zero while their upper production limits are set equal to the total load of the power system or higher. Under normal operating conditions e.g. no load shedding, this kind of generators are assigned zero load from the optimization model since their operation does not minimize the operating cost of the power system. In addition, those “imaginary” generators are not considered able to provide reserves capacity. However, in case of load shedding situations, when the available on-line units are not able to meet the total demand, these units are dispatched with positive power output as far as the power balance constraints should be met.

During the operation of the power several contingency states may occur between different time steps. In the current report the main focus is given on contingencies resulting from generator outages which occur over sequential time frames e.g. for different time periods t. In any case the methodology may easily be applied also in different kinds of contingencies such as transmission lines faults following the same procedure as described here.

As a first step of the methodology, the above presented model is executed for the normal state of the power system, where no contingencies have occurred. The results of the simultaneous energy and reserves dispatch model provide the main technical and economic parameters of the power system such as the energy dispatch \( P_{i,T,j} \) and the reserves capacity procured for each unit \( RU_{i,T,j} \). Furthermore, the Locational Marginal Prices (LMP) and Ancillary Service Marginal Price (ASMP) are obtained – (Ex-Ante prices).

The constraint of the reserve capacity requirement is formulated such that to take into account the available generating capacity of the power system under the current contingency state. It is obvious that the reserve requirement cannot be higher than the difference between the available generating capacity and the load plus the losses. Therefore, the considered constraint under each contingency state is formulated as:

\[
R_{T,j}^{RU} = \min \left( \max \left( p_{i}^{Max}, \sum_{i \in \text{elol}} p_{i}^{Max} \right), \sum_{i} p_{D_{t,j}} - \sum_{i} p_{T,i,j}^{Loss} \right)
\]
The set $i_{ol}$ includes the on-line generators for the current operational state of the power system.

The next step of the methodology consists of executing an OPF model during the real-time operation of the power system in order to re-dispatch the system and obtain the Ex-Post prices as described previously.

Moreover, before the execution of the OPF model during the real time operation, for each new state, reformulation of some of the problem constraints is necessary in order to be consistent with the previous state of the system. The constraint that has to be reformulated is the maximum generating capacities of the remaining on-line units so as to be aligned with the reserves capacity procurement obtained by the last execution of the simultaneous energy and reserve dispatch model. In this case the considered constrain would take the following form:

$$P_{t,t,j} \leq RU_{t,T-1,j} + P^{Max}_{t,T-1,j}$$

The OPF model used to calculate the Ex-Post prices is summarized in the following equations:

$$\min C^T_{t,j} = \sum_{i=1}^{N-1} C_i(P_{i,t,j}) + C_N\left[P_{N,t,j}(x_{t,j})\right]$$

Subject to:

$$\Delta P_{i,t,j}(x_{t,j}, P_{i,t,j}) = P_i(x_{t,j}) - P_{i,t,j} = 0$$

$$\Delta Q_{i,t,j}(x_{t,j}, Q_{t,j}) = Q_i(x_{t,j}) - Q_{i,t,j} = 0$$

$$P_{i,t,j} \leq RU_{i,T-1,j} + P^{Max}_{i,T-1,j}$$

$$P^{Min}_i - P_{i,t,j} \leq 0$$

$$F_{k,t,j}(x_{t,j}) - F^{Max}_{k,t,j} \leq 0$$

In the above equations the subscript T-1 denotes the last discrete time step when the simultaneous energy and reserves dispatch model was executed as well as the subscript t denotes the real-time operation between T-1 and T.

After the necessary constraint is properly reformulated the OPF model (no reserve capacity dispatch) is executed and updated information about the LMP and ASMP are obtained (Ex-Post prices). Furthermore, it can be examined whether a load shedding situation occurs. The main indicator of load shedding is the non-zero value of power production of the “imaginary” generators as described previously.
In case of load shedding the load shedding value for the customers can be calculated using the results of the OPF. Consideration of further stages resulting from this state is not useful, since it is obvious that they will also result in load shedding situations.

On the other hand if the power system is able to meet the demand under the considered contingency state, the simultaneous energy and reserves dispatch optimization model is executed again taking into account the updated parameters of the power system.

As far as the power system is now optimally re-dispatched both in terms of energy and reserves capacity, the methodology moves to a further contingency state where one more unit is set to be off-line.

The above steps are repeated iteratively for all the different contingency states that may result from the initial setup of the power system.

The procedure described above is presented schematically in the following figure.
Figure 6 Flowchart of contingency analysis methodology

Normal State:
- N units online
- Run simultaneous Energy and Reserves capacity dispatch
- Reserves requirement = largest operating unit

Contingency State:
- N-n units online, n=j
- Set P_{max}(T,j) = P_{L}(T-j)+R(T,j)

Run OPF

If Load Shedding
- YES: Calculate Load Shedding value for customers
- NO: Update Reserves requirement
- Run simultaneous Dispatch Energy and Reserves Capacity
- Obtain LMP and ASMP (Ex-Post)

Move to the next stage
- N=n units online
5. Case Study

5.1 Description of the power system

The methodology presented in the previous part of this report is applied in a small scale power system.

For the implementation of the methodology the MATPOWER package was used. MATPOWER is a package of MATLAB M-files for solving power flow and optimal power flow problems. This toolbox was developed by PSERC at Cornell University. The main advantage of MATPOWER, apart from its computational robustness, is the ability that gives to the user to modify the code. [9]

Using the original MATPOWER’s functions and the “Callback Functions” method, the necessary programming codes were developed for the implementation of the described methodology.

The network topology of the power system that was used for the demonstration purposes of this report is presented in following figure. The current setup of this power system is provided by the MATPOWER library (case9.m). [9]

![Network topology of the case study power system](image)

The power system consists of 9 buses, 3 loads and 3 generators. Furthermore, the different buses are connected with 9 branches. The following tables provide the main characteristics of the components of the studied power system.
From Bus | To Bus | r (p.u.) | x (p.u.) | b (p.u.) | Rate (MVA)
---|---|---|---|---|---
1 | 4 | 0 | 0.0576 | 0 | 250
4 | 5 | 0.017 | 0.092 | 0.158 | 250
5 | 6 | 0.039 | 0.17 | 0.358 | 150
3 | 6 | 0 | 0.0586 | 0 | 300
6 | 7 | 0.0119 | 0.1008 | 0.209 | 150
7 | 8 | 0.0085 | 0.072 | 0.149 | 250
8 | 2 | 0 | 0.0625 | 0 | 250
8 | 9 | 0.032 | 0.161 | 0.306 | 250
9 | 4 | 0.01 | 0.085 | 0.176 | 250

Table 1 Branch Data

All the values given in Table 1 Branch Data are in p.u. using as base 100 MVA. The symbols r, x and b denote the resistance, the reactance and the susceptance for each branch respectively. The value of “Rate” denotes the maximum power that can be transferred through each branch.

| Bus | Qmax (MVar) | Qmin (MVar) | Pmax (MW) | Pmin (MW) |
---|---|---|---|---|
1 | 300 | -300 | 250 | 10 |
2 | 300 | -300 | 300 | 10 |
3 | 300 | -300 | 270 | 10 |

Table 2 Generator data

| Bus | P (MW) | Q (MVar) |
---|---|---|
5 | 40 | 30 |
7 | 60 | 35 |
9 | 80 | 50 |

Table 3 Load Data

The values of active and reactive power limits for each generator as well as the values of load at the different buses of the power system are given in MW and MVar respectively.

Table 4 provides the parameters which define the generator cost function for each unit in the form of 2nd order polynomial cost. The cost function has the form:

\[ f(p) = c_2 p^2 + c_1 p + c_0 \]
Where $f$ and $p$ are $$/hr$ and MW respectively.

<table>
<thead>
<tr>
<th>Bus</th>
<th>C2</th>
<th>C1</th>
<th>C0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>0.085</td>
<td>1.2</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>0.1225</td>
<td>1</td>
<td>335</td>
</tr>
</tbody>
</table>

Table 4 Generator Cost Data

The following figures present the operating cost ($$/hr$) and the incremental cost ($$/MWh$$) for the three generating units of the power system, according to the data provided in Table 4.

Figure 8 Operating costs in $$/hr$$ for three generating units in respect to their nominal capacity

Figure 9 Incremental costs in $$/MWh$$ for three generating units in respect to their capacity
For demonstration purposes the regulation – up reserves prices were set equal to 10, 15 and 20 $/MW for generators 1 to 3 respectively. Moreover, it was assumed that all the generators belong to the same zone as well as the maximum regulation-up capacity that are able to provide equals their maximum nominal power output.

For simplicity reasons and without loss of generality it was further assumed that the load of the power system remains constant in respect to time t and between the different stages i.e. contingency states. In reality this value is expected to be changed for the different stages as the studied phenomenon is developing in time.

Finally, under the current study it is considered that the generating units are following the same cost function regardless if they provide energy or reserves. In the general case it can be argued that a generator may have different cost functions when it provides energy compared to the situation its reserve capacity is actually used.

5.2 Results

This part of the report provides the results of the methodology described and implemented in the power system presented in the previous paragraph.

Figure 10 is a schematic representation of the different stages and states of the power system under investigation according to the followed methodology for contingency analysis. Each node in Figure 10 corresponds to a different operational situation of the power system where different contingencies have occurred. In particular the number written in the each node denotes the respective number of the generating unit which is considered to be off-line due to outage. Furthermore, the numbers in the boxes next to each node are used to identify the different operating conditions and facilitate the presentation of the results.

As it can be seen from Figure 10, in the current case study there are 2 stages resulting from N-1 generating units (N=3). It is obvious that proceeding to further stages, where N units would be offline, would not produce any useful results since no percentage would be covered.
The methodology starts by executing the simultaneous energy and reserve dispatch model under the normal operating conditions of the power system (NS – in Figure 10).

The following tables present the main results for the normal operating condition of the power system, as they are produced from MATPOWER.
### Bus Data

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Mag(pu)</th>
<th>Ang(deg)</th>
<th>Generation P (kW)</th>
<th>Q (kVAR)</th>
<th>Load P (MW)</th>
<th>Q (kVAR)</th>
<th>Lambda(S/MVA-hr)</th>
<th>E</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.094</td>
<td>0.000</td>
<td>30.37</td>
<td>2.28</td>
<td>-</td>
<td>-</td>
<td>16.682</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1.095</td>
<td>4.799</td>
<td>88.72</td>
<td>-6.99</td>
<td>-</td>
<td>-</td>
<td>16.282</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>1.085</td>
<td>3.892</td>
<td>62.39</td>
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<td>-</td>
<td>-</td>
<td>16.285</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>1.093</td>
<td>-0.838</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.682</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
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<td>1.090</td>
<td>-0.943</td>
<td>-</td>
<td>-</td>
<td>40.00</td>
<td>30.00</td>
<td>16.693</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.100</td>
<td>2.137</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.285</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>1.092</td>
<td>0.880</td>
<td>-</td>
<td>-</td>
<td>60.00</td>
<td>35.00</td>
<td>16.365</td>
<td>0.021</td>
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</tr>
<tr>
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<td>1.100</td>
<td>2.159</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.282</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1.076</td>
<td>-1.884</td>
<td>-</td>
<td>-</td>
<td>80.00</td>
<td>50.00</td>
<td>16.763</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td></td>
<td>131.48</td>
<td>-31.82</td>
<td>180.00</td>
<td>115.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Branch Data

<table>
<thead>
<tr>
<th>Branch #</th>
<th>From Bus</th>
<th>To Bus</th>
<th>From Bus Injection P (MVA)</th>
<th>Q (MVAR)</th>
<th>To Bus Injection P (MVA)</th>
<th>Q (MVAR)</th>
<th>Loss (I^2 * Z)</th>
<th>E (MVAr)</th>
<th>Q (MVAr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>30.37</td>
<td>2.28</td>
<td>-30.37</td>
<td>-1.83</td>
<td>0.000</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>2.87</td>
<td>-6.74</td>
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<td>-12.08</td>
<td>0.002</td>
<td>0.01</td>
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</tr>
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<td>6</td>
<td>-37.13</td>
<td>-17.92</td>
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<td>0.456</td>
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</tr>
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<td>0.000</td>
<td>2.30</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>24.80</td>
<td>-6.37</td>
<td>-24.73</td>
<td>-18.19</td>
<td>0.064</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>9</td>
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<td>-16.81</td>
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<td>-0.28</td>
<td>0.093</td>
<td>0.79</td>
<td></td>
</tr>
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<td>7</td>
<td>8</td>
<td>2</td>
<td>-88.72</td>
<td>11.12</td>
<td>88.72</td>
<td>-6.99</td>
<td>0.000</td>
<td>4.13</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>9</td>
<td>53.36</td>
<td>-10.53</td>
<td>-52.59</td>
<td>-21.53</td>
<td>0.769</td>
<td>3.87</td>
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</tr>
<tr>
<td>9</td>
<td>9</td>
<td>4</td>
<td>-27.41</td>
<td>-28.47</td>
<td>27.50</td>
<td>8.57</td>
<td>0.094</td>
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<td></td>
</tr>
<tr>
<td>Total:</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>1.476</td>
<td>14.88</td>
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### Voltage Constraints

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Vmin (pu)</th>
<th>Vmin</th>
<th></th>
<th>Vmax</th>
<th></th>
<th>Vmax (pu)</th>
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<tbody>
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<td>6</td>
<td>-</td>
<td>0.900</td>
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<td>8</td>
<td>-</td>
<td>0.900</td>
<td>1.100</td>
<td>1.100</td>
<td>21.918</td>
<td></td>
</tr>
</tbody>
</table>
Table 5 Results for normal operating condition of the power system

From the numerical values presented in the table above it can be seen the load assigned to each generator, which is equal to the total demand of 180 MW plus the real power losses which account for 1.478 MW.

Furthermore, as it was expected under the normal operating condition, there is no load shedding. This can be verified from the corresponding values of LMPs in the different buses, which are far below the extreme value of VOLL that is assigned to the “imaginary” generators (GEN 4, 5 & 6) as described previously. In particular the LMPs are taking values in the range of 16.28 $/MWh at bus 2 to 16.76 $/MWh at bus 9.
No transmission line capacity constraints are violated. However, the voltage magnitude at buses 6 and 8 has reached its upper limit (1.1 p.u.).

According to the criterion determined in the methodology regarding the reserves requirement, in this case the regulation-up reserve requirement was set equal to 300 MW, which is the nominal capacity of the largest operating unit (Unit 2). The algorithm optimizes the reserve capacity allocation to the different units according to their corresponding cost. As a result, 219.63 MW and 80.37 MW of reserve capacity are allocated to units 1 and 2 respectively. It can be seen that the largest amount of reserve capacity is assigned to Unit 1, which has the lower reserve capacity cost (10 $/MW), while the rest of the needed reserve capacity is given to Unit 2 with cost of (15 $/MW). Unit 3 which faces the highest reserve capacity cost is not assigned with any reserve capacity. Therefore, the ASMP in this case is 15 $/MW and it is equal to the corresponding cost of Unit 2.

The following tables present the most important results after the execution of the methodology on the considered power system, in respect to the number of each node as provided in Figure 10.

<table>
<thead>
<tr>
<th># Node</th>
<th>Bus 1</th>
<th>Bus 2</th>
<th>Bus 3</th>
<th>Bus 4</th>
<th>Bus 5</th>
<th>Bus 6</th>
<th>Bus 7</th>
<th>Bus 8</th>
<th>Bus 9</th>
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<tr>
<td>1</td>
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<td>22.59</td>
<td>22.69</td>
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</tr>
<tr>
<td>2</td>
<td>47.27</td>
<td>49.04</td>
<td>49.05</td>
<td>47.27</td>
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<td>49.05</td>
<td>49.25</td>
<td>49.04</td>
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</tr>
<tr>
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<td>40.50</td>
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<td>29.58</td>
<td>29.67</td>
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<td>5</td>
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<td>49.05</td>
<td>47.27</td>
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<td>9</td>
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<td>38.66</td>
<td>39.52</td>
<td>40.52</td>
<td>40.52</td>
<td>39.52</td>
<td>39.22</td>
<td>38.66</td>
<td>40.53</td>
</tr>
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</table>

Table 6 LMPs in $/MWh for all the buses of the power system and for each different node of Figure 10

By inspection of Table 6 it can be easily concluded that the power system faced only one load shedding situation as depicted by the LMPs of node 8. In the rest of the cases the on-line generation capacity and the reserves were able to compensate adequately the contingencies resulting from the outage of the other units.

However, it should be noted that the LMPs are significantly increased compared to the respective values of the normal state. This is an expected result since in any of the cases presented above; the generating units that remain online have to increase their output to cover the system’s demand. Therefore, given that the cost functions of the generators are strictly increasing the resulting LMPs are increased.
Furthermore, it should be mentioned that the resulting LMPs present high volatility between the different nodes of the solution procedure. This behavior can be depicted better in the following figure which presents the value of LMP in each bus for all the different nodes (different lines).

![Figure 11: LMPs in $/MWh for all the buses of the power system and for each different node](image)

From the previous figure it can be concluded that the highest LMPs occur at nodes 5 and 6, which both have the same parent node 4 at which Unit 2 is set off-line. On the other hand, the lower LMPs are in nodes 1 and 7, where unit 3 and 1 are set off-line respectively.

This phenomenon can be interpreted by looking at the operating cost and incremental cost functions of Figure 8 and Figure 9. Unit 2 has always the lowest incremental cost while it has also the lowest operating cost for the power production region above 90 MW. In the meantime, Units 1 and 3 have significantly higher incremental costs, while their operational cost functions are very similar.

Therefore, in cases when Unit 2 is out of order, the demand of the power system has to be covered from the remaining units 1 and 3 which yield to higher LMPs. Furthermore, in each of the latter cases the lowest LMP is produced in the bus where the remaining on-line generator operates, since it depicts only the cost of energy as far as the specific buses have zero transmission losses and congestion LMP components.

In the contrary, at nodes 1 and 7 where either Units 1 or 3 are off-line and Unit 2 is operating normally the values of LMP are significantly lower. This can be verified also
by the following table which presents the load assigned to each power generating unit for the different nodes.

<table>
<thead>
<tr>
<th># Node</th>
<th>Gen 1</th>
<th>Gen 2</th>
<th>Gen 3</th>
<th>Gen 4</th>
<th>Gen 5</th>
<th>Gen 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57.68</td>
<td>123.83</td>
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<td>0.00</td>
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<td>0.00</td>
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<tr>
<td>3</td>
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<td>0.00</td>
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</tr>
<tr>
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<td>0.00</td>
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<td>0.57</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 7 Load assigned (MW) to power generating units for the different nodes

According to Table 7 in every node where Unit 2 is on-line it has the highest energy production due to its low operating and incremental costs. Furthermore, it should be noted that in all nodes, apart from node 8, the “imaginary” Units 4 to 6 have zero load. As already mentioned, node 8 depicts the single situation when load shedding occurs and as it can be seen in this case Units 4,5 and 6 have positive generation. Finally, Table 7 clearly shows how the different contingencies affect also the real power losses of the power system. In each case the real power losses can be calculated as the sum of generation of the on-line units minus the demand (180 MW). It is evident that the real power losses are higher compared to the normal operating state.

The following table presents the reserve capacity assigned to each generator for the all the different nodes of the solution procedure. In every node the sum of the reserve capacity that all the on-line generators provide is equal to reserve requirement which is calculated according to the criterion described and it is presented in the last column of Table 8.

It should be mentioned that for all the nodes of the first stage of the solution procedure e.g. nodes 1,4 and 7, the reserve requirement is equal to largest operating unit. On the other hand, at the nodes of stage 2 the reserves requirement is dictated by the remaining available generating capacity of the power system.

In addition, the results presented in the following table verify the accuracy of the followed methodology since it is obvious that the reserves capacity is optimally allocated between the different generators according to their price. For example, GEN 1, which faces the lowest regulation-up cost, is assigned with the highest value of reserves capacity in all the nodes where it is on-line.
Table 8 Reserves requirement and reserve capacity of each generator for the different nodes

Table 9 provides the ASMP for the different nodes of the solution. It can be observed that the ASMP takes different value for each node, which is affected by the available on-line units, their reserve capacity and the reserve requirement for every operating state of the power system.

In cases where the total reserve capacity of the on-line units is greater than the reserve requirement in this particular node, the ASMP is equal to the highest marginal cost of reserves of the generators provide positive reserve capacity. On the other hand, in cases when the constraint refers to the total amount of energy plus reserves to be lower or equal to the capacity of each unit, then the ASMP reflects the “scarcity rent” of the generator regarding both the energy and the reserves.

For example, the ASMP = 12.0312 at node 2 equals the marginal cost of reserves from GEN 1 (10 $/MW) plus the difference of the LMP = 47.27 $/MWh at bus 1 minus the incremental operating cost of GEN 1 (45.23 $/MWh) at 182.9 MW as it is calculated by the corresponding incremental cost function.
The load shedding situation which occurred in node 8 is a result of the inability of the remaining on-line GEN 3 to cover the demand, taking into account its power production capability defined as the sum of energy production and reserve capacity dispatched by the algorithm.

In particular, node 8 is an offspring of node 7 where 87.08 MW of energy and 95.37 were dispatched at GEN 3. Furthermore, at node 7 GEN 2, which is still on-line, is assigned with the largest reserve capacity equal to 204.63 MW, as a result of its high nominal capacity and low reserve cost. Therefore, as long as GEN 2 is set off-line at node 8, the power system losses significant reserves capacity and as a result GEN 3 is not able to cover the total demand and the corresponding real losses. This situation leads to positive dispatch of “imaginary” generators 4, 5 and 6 which corresponds to equal load shedding.

This situation reveals the importance of contingency analysis of the power system in order to obtain important information about its ability to serve the customers under different operating conditions. In addition, this kind of studies can reveal significant parameters that influence the secure and economically efficient operation of the power system in a deregulated energy market.

This study shows that there are numerous different technical and economic parameters which determine the ability of the power system to serve the demand with optimal cost. For example, the above described load shedding situation of node 8 could be avoided if GEN 3 had lower reserve cost and therefore during the last optimal energy and reserves dispatch it would have been assigned with higher reserves capacity. In this case GEN 3 might be able to cover the total system’s demand despite the outage of generators 1 and 2.

Furthermore, one other very important technical parameter that influences the operation of the power system under contingencies is its existing network topology and maximum transmission capacities of the lines. In the current case study all the three buses where the generators are placed are connected with a single branch with their neighboring buses. This may lead to inability of the system to serve properly the demand in cases when, even if it exists adequate generating capacity, the corresponding transmission lines are reaching their upper capacity limit.
6. Conclusion

This project intended to present a new market design where a 5-min simultaneous energy and reserve pre-dispatch model and re-dispatch model for the real operating time of the power system are used.

The study provided all the necessary theoretical background and the steps of the methodology followed in order to evaluate the technical and economic efficiency of a power system using the proposed market scheme under different contingency states.

The implementation of this work in a small scale power system provided very interesting results underlying the volatility of the LMPs and ASMPs in respect to the different operational conditions of the power system. Furthermore, it proved the importance of the results for the TSO so as to take all the necessary measures in order to mitigate the load shedding situations.

7. Future work

Based on the current methodology described in this report significant opportunities for future further development are given. A more thorough and complete study of the proposed market structure in combination with system’s reliability assessment indices may be pursued.

The presented contingency analysis methodology may be further developed into a stochastic programming optimization problem where some representative indices of the power system reliability are included e.g. EFOR (Equivalent Forced Outage Rate).

As a result, the optimization problem can be augmented in order to account also for the minimization of some reliability factors of the power system such as the Loss Of Load Probability (LOLP) and the Expected Energy Not Supplied (EENS).
8. References

2. Lectures Power System Operation (2011), Ass. Prof. Yi Ding, CET – DTU
8. Reliability assessment of restructured power systems using optimal load shedding technique (2008), P. Wang, Yi Ding, L. Goel, IET Generation, Transmission & Distribution

9. Acknowledgements

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